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Engineering Notes

ENGINEERING NOTES are short manuscripts describing new developments or important results of a preliminary nature. These Notes cannot exceed six manuscript pages and three figures; a page of text may be substituted for a figure and vice versa. After informal review by the editors, they may be published within a few months of the date of receipt. Style requirements are the same as for regular contributions (see inside back cover).

Effect of Kinetic Energy Projectile Inertia upon Precision

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Nomenclature

C_D = drag coefficient
$C_{L\alpha}$ = lift coefficient
$C_{M\alpha}$ = moment coefficient
D = projectile diameter
I_t = transverse moment of inertia
L = downrange target distance
m = projectile mass
t_e = time of shot exit
u, v = transverse components of projectile velocity at launch
V_m = projectile muzzle velocity
x_t, y_t = horizontal and vertical target impacts relative to aim point
z_0 = initial displacement
Θ = aerodynamic jump
ξ_0' = launch angular rate (rad/caliber of projectile travel), $= \beta_0' + i\alpha_0'$

Introduction

PRECISION, or round-to-round dispersion, is important to the accuracy of tank gunnery and, as such, is an acceptance specification for ammunition. Both the actual round and its companion training projectile must meet similar criteria. The principal tank cartridge is the kinetic energy penetrator, a fin-stabilized, long rod fabricated from a dense metal such as a tungsten alloy. Training rounds must closely resemble the actual cartridge in terms of visual appearance, handling characteristics, size, weight, and, up to a point, ballistic performance. In particular, it must remain within the boundaries of military reservations; thus, maximum range is specified to be a small fraction of that of the actual round. Currently, the kinetic energy training round is a flare-stabilized projectile with a steel core. The round provides reasonable ballistic similitude out to 3 km. Beyond this range, the high drag of the flare causes rapid trajectory decay limiting the maximum range. The muzzle velocity is 1700 m/s giving the steel core an appreciable penetration capability.

To improve range safety, Kennedy et al.[1] attempted to provide an alternative with greatly reduced penetration. By employing a hollow aluminum flight body with fin stabilization, they succeeded in matching the trajectory of the existing trainer, while reducing the penetration by about a factor of 10. However, round-to-round dispersion grew more than three times. The authors speculated that an increase in aerodynamic jump caused this result; however, attempts to decrease the aerodynamic jump coefficient by altering the projectile center-of-gravity location were only partially successful. The present paper examines the launch dynamics of these projectiles and correlates their inertial properties with both overall precision and the components of flight disturbance as identified by Lyon et al.[2] and Murphy.[3]

Experiment

Three rounds types are considered: tungsten alloy (WA), steel, and hollow aluminum. All are fin stabilized and fired from the 120-mm, M256 cannon. The tungsten alloy round is the German DM13. The steel round is the XM866, an experimental fin-stabilized training with a steel core. The hollow aluminum projectile is the candidate of interest and has the same outer geometry as the XM866. All rounds are launched encased in nearly identical sabots. Tests are fired from different gun tubes, at different sites, and at different times. Although these factors influence accuracy, it is hypothesized that the effect on precision is not great. Simply stated, the tank error budget is treated as arising from three, independent sources: tank-to-tank bias, occasion-to-occasion bias, and round-to-round dispersion. The differences in the test conditions would affect the bias, but are assumed to have limited influence on dispersion.

Data are taken in two separate experiments. The DM13 and XM866 are fired at the Army Research Laboratory (ARL) Transonic Range using laboratory diagnostics.[2] The aluminum round is fired at Yuma Proving Ground using standard lot acceptance procedures. The instrumentation at the Transonic Range consists of six orthogonal flash radiograph stations over the first 15 m of flight, 25 orthogonal spark shadowgraph stations over 250 m, and an impact target at 1 km. The Yuma data set is sparser. Six yaw cards cover the first 100 m of flight, and impact targets are located at 1, 2, and 3 km. An epicyclic fit is made to the yaw data and extrapolated back to the muzzle for an estimate of launch angular rate. Accuracy of the flash radiographs and spark shadowgraphs is 0.1 deg, while the yaw cards is 0.25 deg, producing a one-sigma error in the estimated launch angular rate of 0.1 and 0.3 rad/s, respectively, compared to a mean rate of 5 rad/s. The targets provide the difference between the aim and impact points, that is, total jump. Target impact is measured to within 50 mm for a one-sigma trajectory error of 0.02 mrad compared to a mean jump of 0.5 mrad.

The aerodynamic and inertial properties of the tungsten alloy and steel rounds were measured at the ARL Transonic Range, whereas those of the hollow aluminum round were computed using PRODAS[4] (Table 1).

From the measured angular rate and total jump, it is possible to calculate the aerodynamic jump and infer transverse linear velocity at launch. Aerodynamic jump is expressed[3] as

$$\Theta = (I_t/mD^2)(C_{L\alpha}/C_{M\alpha})\xi_0' \tag{1}$$

Subtracting this from the total jump yields the initial projectile velocity vector:

$$(u + iv)/V_m = (x_t - iy_t)/L - \left(I_t/mD^2\right)(C_{L\alpha}/C_{M\alpha})(\beta_0' + i\alpha_0') \tag{2}$$

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Table 1 Properties of fin-stabilized projectiles

Projectile nomenclature	m, kg	D, m	I_t, kg-m^2	V_m, m/s	C_D	$C_{L\alpha}$	$C_{M\alpha}$
DM13 (WA)	4.43	0.038	0.048	1650	0.322	7.58	−16.9
XM866 (steel)	2.73	0.038	0.034	1680	0.314	7.20	−14.2
Hollow Al	0.94	0.038	0.0074	1700	0.269	8.02	−7.00

Table 2 Circular probable errors for various rounds normalized to value for tungsten

Projectile nomenclature	CEP $u, v/V_m$	CEP ξ_0'	Aerodynamic jump coefficient, Θ/ξ_0'	CEP on target
DM13 (WA)	1	1	1	1
M866 (steel)	2.23	2.53	1.31	1.24
Aluminum	5.02	4.13	1.84	3.56

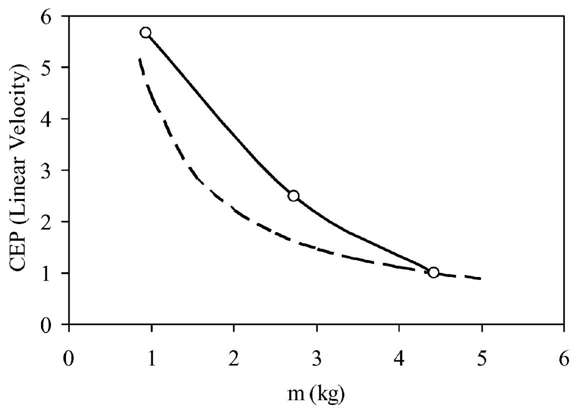

Fig. 1 Plot of flight mass vs CEP of linear velocity (○, measured CEP of initial linear velocity and – – –, m_{WA}/m).

Analysis

All firings show structural integrity and produce first maximum yaw levels of less than 2 deg, generally indicative of a good launch. It is of interest to compare the variability of initial conditions with the distributions of fall of shot. Dispersion is typically different in the horizontal and vertical directions. For clarity, the approach of Grubbs[5] is used, which defines the equivalent circular error probable (CEP) as the root sum squares of the two components. Test results are summarized in Table 2 where all values are normalized to those of the tungsten alloy round. Lateral velocity components, u and v, are normalized by the muzzle velocity.

The growth in CEP from tungsten to aluminum is apparent. A number of factors are responsible. The linear and angular velocities both take on a progressively more random nature. In addition, the aerodynamic jump coefficient is roughly twice as large for the aluminum round as for the tungsten round serving to amplify the effects of initial angular velocity disturbances. The fact that the dispersion on target does not grow to the same extent as the CEPs in linear and angular velocities reflects the fact that the two can interact in a fashion to partially cancel each other.

The CEPs are correlated with the inertial properties of the projectiles in Figs. 1 and 2. Because the sabots are nearly identical for all three rounds, their mass and moment of inertia are not considered in the comparisons. The CEP in initial lateral velocity decreases with increasing projectile mass, while the CEP in initial angular rate decreases with increasing transverse moment of inertia. Reversing the variables, for example, correlating linear velocity with moment of inertia from Table 2, produces a less satisfying result.

Launch disturbances are associated with gun tube recoiling motion, projectile motion relative to the gun, and sabot discard. Plostins et al.[6] show that gun motion is relatively consistent from shot to shot; however, they find that projectile dynamics are quite variable. The major source of transverse perturbation originates in the gun as a result of vibration of the projectile within the elastic sabot.

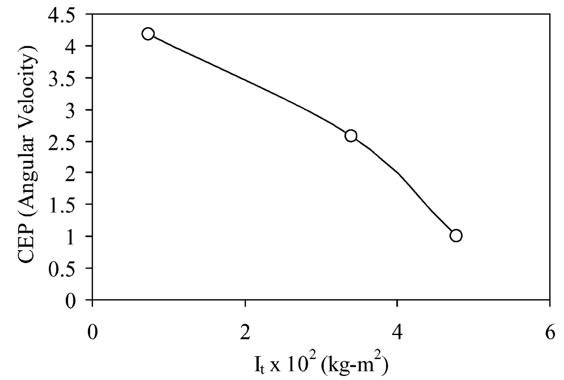

Fig. 2 Plot of transverse moment of inertia vs normalized CEP of angular velocity.

Even sabot discard perturbations can be traced to the initial state of the multibody dynamics following elastic decompression from the gun. Many analyses[4,7] of in-bore vibration treat the gun, sabot, and projectile as a set of lumped mass points connected by a series of springs. To obtain an insight into the current data, a model of a one-dimensional spring mass provides a simple analog of lateral, in-bore vibration of a projectile within the sabot.

The solution for an undamped oscillator responding to an initial displacement is $z = z_0 \cos(k/m)^{1/2} t$ with derivatives evaluated at a given time t_e as $dz/dt = -z_0(k/m)^{1/2}\sin(k/m)^{1/2}t_e$, and $d^2z/dt^2 = -z_0(k/m)\cos(k/m)^{1/2}t_e$. For a given oscillator, both the initial displacement z_0 and observation time t_e can vary. With the projectile analog, velocity is of interest, and the second derivative gives a measure of the sensitivity of the first derivative to changes in z_0 and t_e. Because the sabots are nearly identical, the variability in initial orientation relative to the gun tube, or z_0, should be equivalent across the cases. Also, the similarity of muzzle velocities is taken to imply a reasonably consistent variability in t_e. This implies that differences in CEP of linear velocity between round types should be associated with the coefficient of the second derivative, the stiffness to mass ratio k/m.

For the DM13, XM866, and hollow aluminum rounds, the sabots provide nearly identical stiffness k; however, each projectile has a significantly different mass; thus, the analog would suggest that linear velocity variability from round to round is proportional to $1/m$. This variation, normalized by the tungsten projectile mass, is plotted in Fig. 1. It is seen that the CEP and mass ratio from the simple analog have similar range and behavior, suggesting that reduction in the stiffness (parameter k) of the sabot could have improved the CEP of the lower mass rounds. This approach was not considered at the time. The rounds were fired with existing sabots that were compatible with the high mass projectiles.

Conclusions

To develop improved training rounds, aluminum was substituted for existing steel bodies. Accuracy firings showed that lower density projectiles had significantly greater round-to-round dispersion than standard designs. First ascribed to larger values of the aerodynamic jump coefficient Θ/ξ_0', careful analysis of the data indicates that variability in initial dynamics dominates. A simple argument suggests that when the inertial properties of the round are changed it is necessary to match the sabot properties, for example, stiffness. To better understand this behavior, higher-fidelity simulations of the in-bore vibration and disengagement dynamics are required.

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